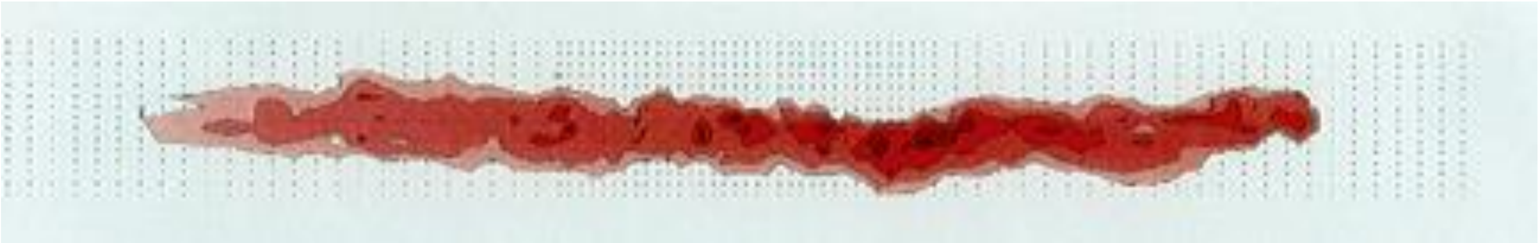
Kriging for Drop Test Data

What is the coverage of these retardant drops?

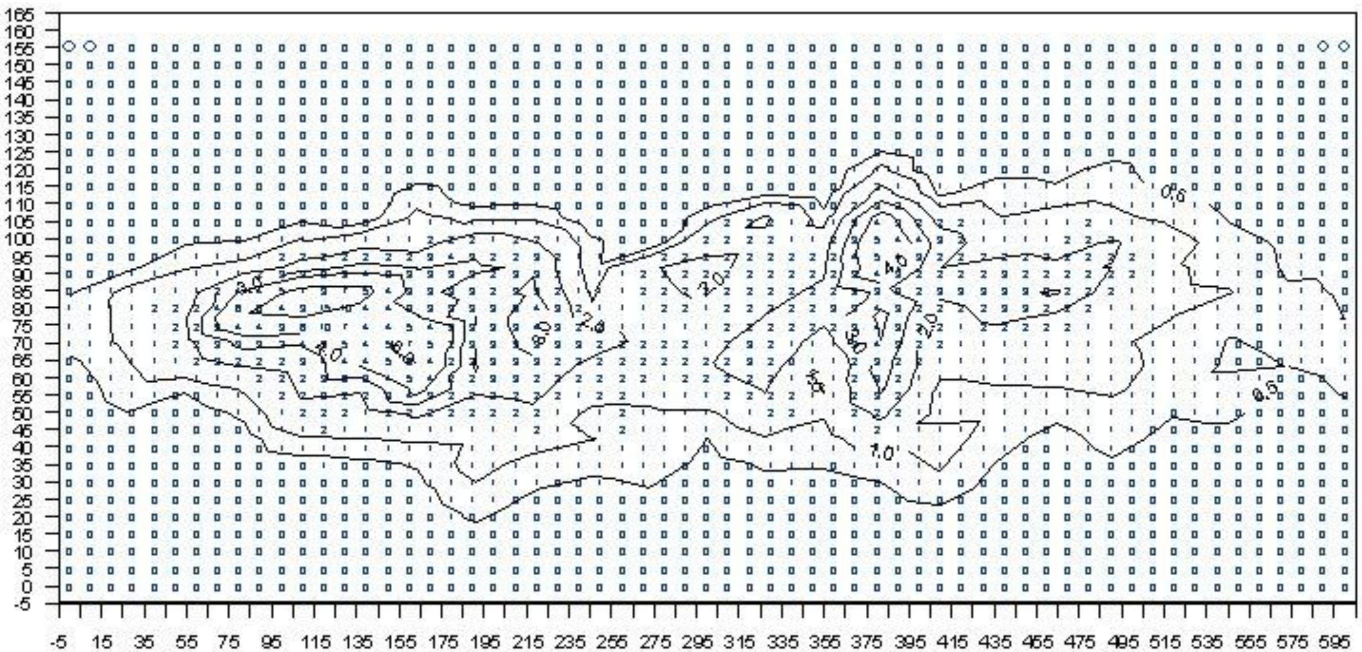
We use the drop test to measure coverage.

Retardant is collected in cups. Cups are capped and weighed.

Knowing the weights allows us to recreate the pattern as a contour plot.



Each cup becomes a gpc value, grams per hundred square feet.



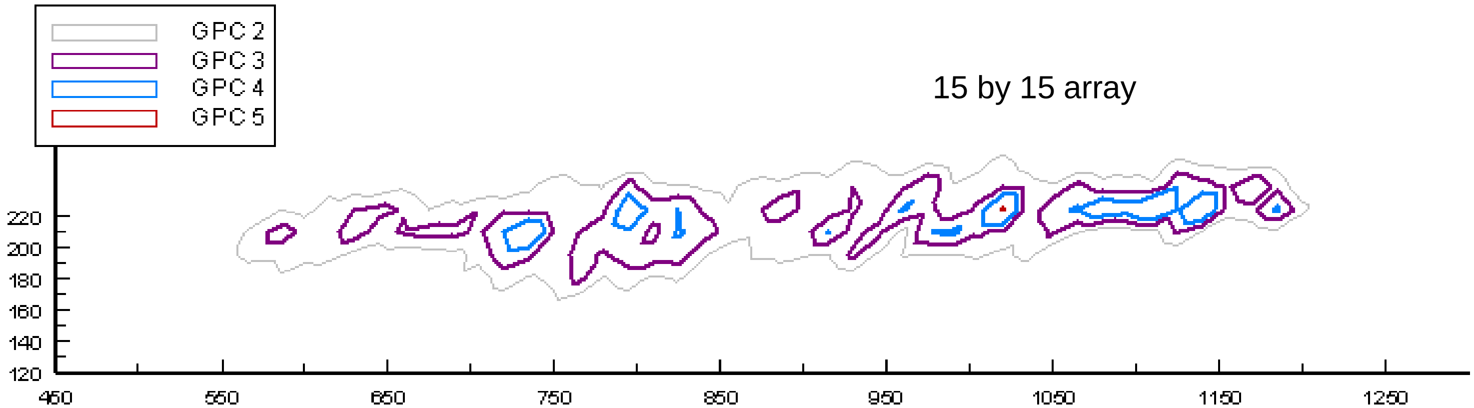
Converting grams of retardant into gpc.

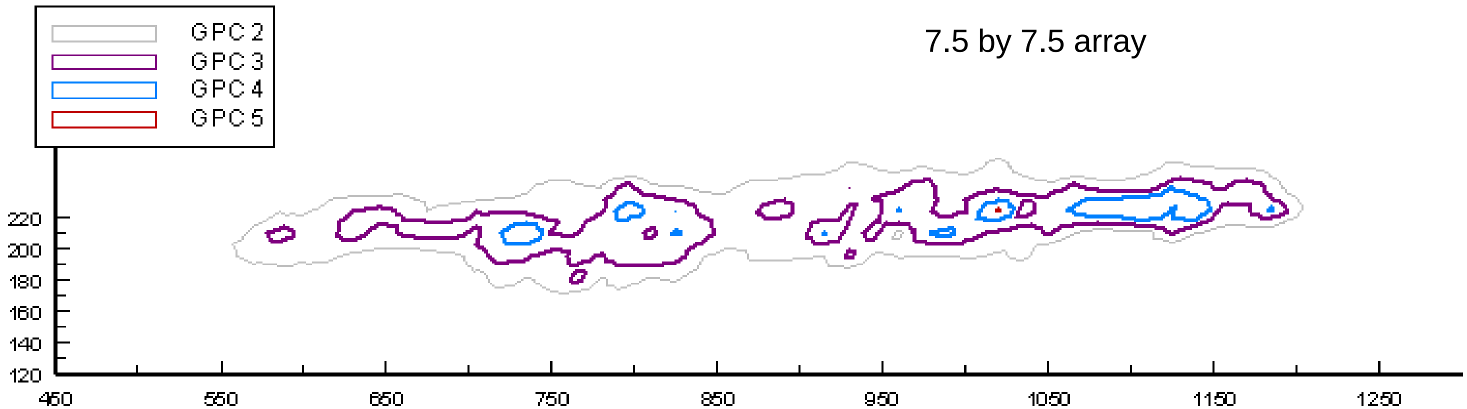
$$\left(\frac{1\text{ gallon}}{3785.4118\text{ cc}}\right)\times\left(\frac{1\text{ grid cup}}{0.001963495\text{ hundred square feet}}\right)=0.13454172$$

$$0.13454172\times\left(\frac{\text{weight of retardant (grams)}}{\text{density of retardant (grams/cc)}}\right)=\text{gpc}$$

Both plots below use the same raw data.

In the first plot, the contours are created from an array of raw data at a 15 by 15 spacing and by interpolation routines inherent in Splus software. In the second plot the points between raw data are estimated using kriging to produce an array of higher resolution (7.5 by 7.5) and then by Splus software.





Which one is more accurate?

We use a weighted average of the raw gpc values to predict unknown values.

We use a method called kriging to come up with weights for the weighted average.

These kriging weights are based on distance, direction and variability of the raw data.

Steps to find the weighted average:

1. Create a variogram which is a mathematical description of the variation between observed gpc values.

2. Use the variogram to estimate the covariance at different distances.

3. Use the covariances to calculate kriging weights.

4. Calculate the weighted average of the raw data.

Step 1: Create a variogram which is a mathematical description of the variation between observed gpc values.

The variogram is the average squared difference between pairs of gpc values.

The table on the next slide shows the variogram values for drop 104.

In the first line of the table we see that at a distance of 23.97096 feet there are 4,120 pairs of cups.
The average squared difference between all these pairs is 0.145876.

distance	gamma (variogram)	number of pairs
23.97096	0.145876	4120
58.1154	0.294227	18028
86.77792	0.334119	11831
111.9615	0.481884	30293
145.6326	0.553698	29346
170.1047	0.613125	34471
201.5029	0.679907	42091
227.975	0.694383	41259
257.2455	0.781188	50939
283.3795	0.771159	42779
310.9756	0.875735	61842
342.395	0.889409	60465
369.5541	0.928662	61478
400.3173	0.975992	67232
427.0981	0.987465	60637
456.3625	1.044142	72292
484.288	1.049219	63810
512.6638	0.948816	75918
542.5489	0.911984	68710
569.6804	0.808484	73019

Note: The distances in this table are calculated from the length of the drop pattern and the grid spacing.

We plot the data from the table and fit a curve.

distance	gamma (variogram)
23.97096	0.145876
58.1154	0.294227
86.77792	0.334119
111.9615	0.481884
145.6326	0.553698
170.1047	0.613125
201.5029	0.679907
227.975	0.694383
257.2455	0.781188
283.3795	0.771159
310.9756	0.875735
342.395	0.889409
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456.3625	1.044142
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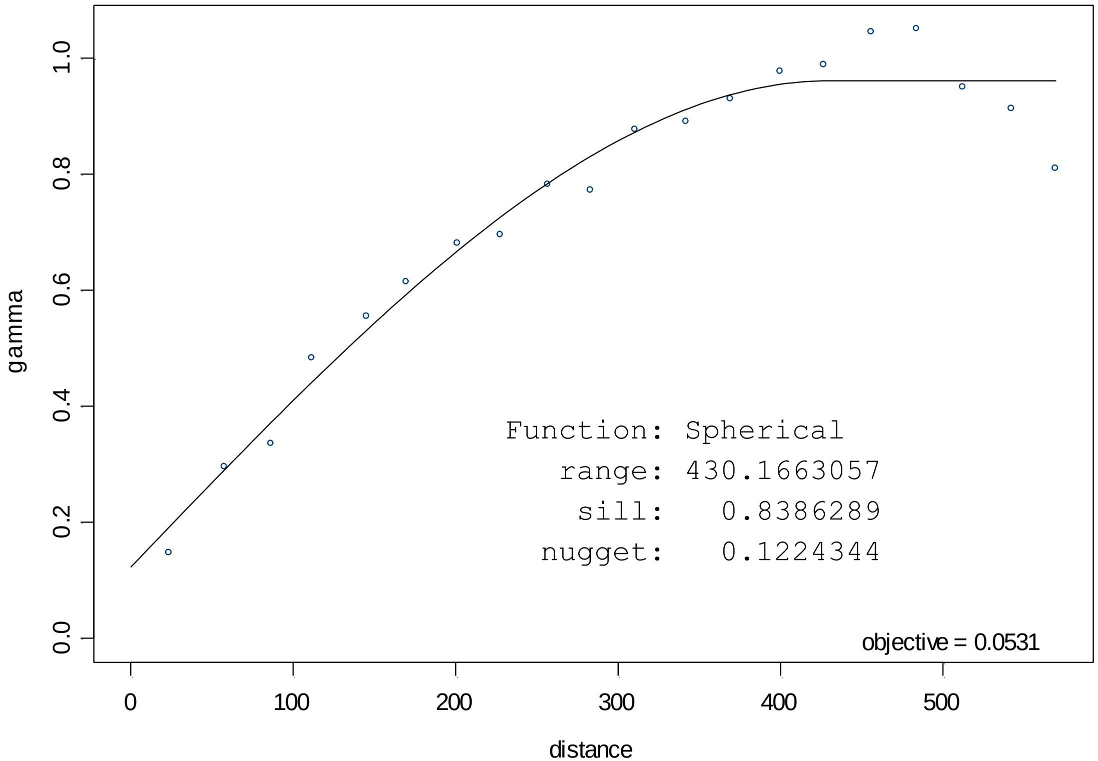
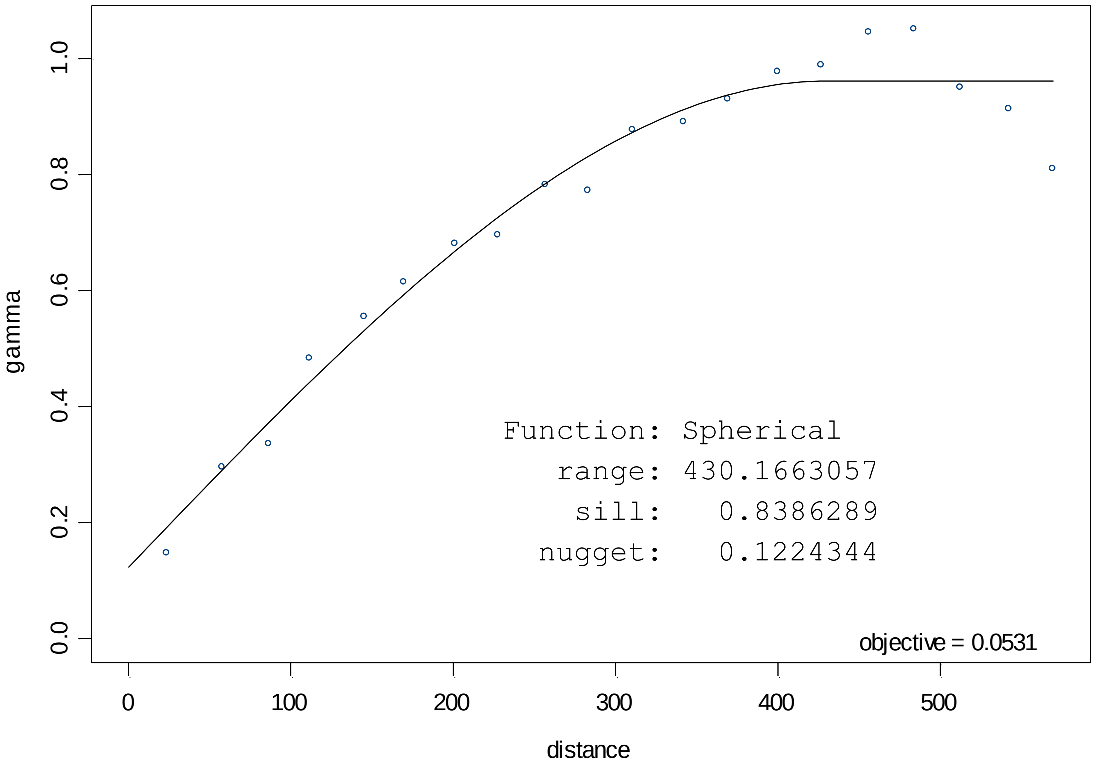


Three features of the trend line are used to calculate weights in our weighted average.

- <u>Range</u>: As the distance between pairs increases, the average squared difference between pairs will increase. Eventually, an increase in distance will no longer produce and increase in average squared difference, and the variogram will reach a plateau. The distance at which the plateau is reached is the range.

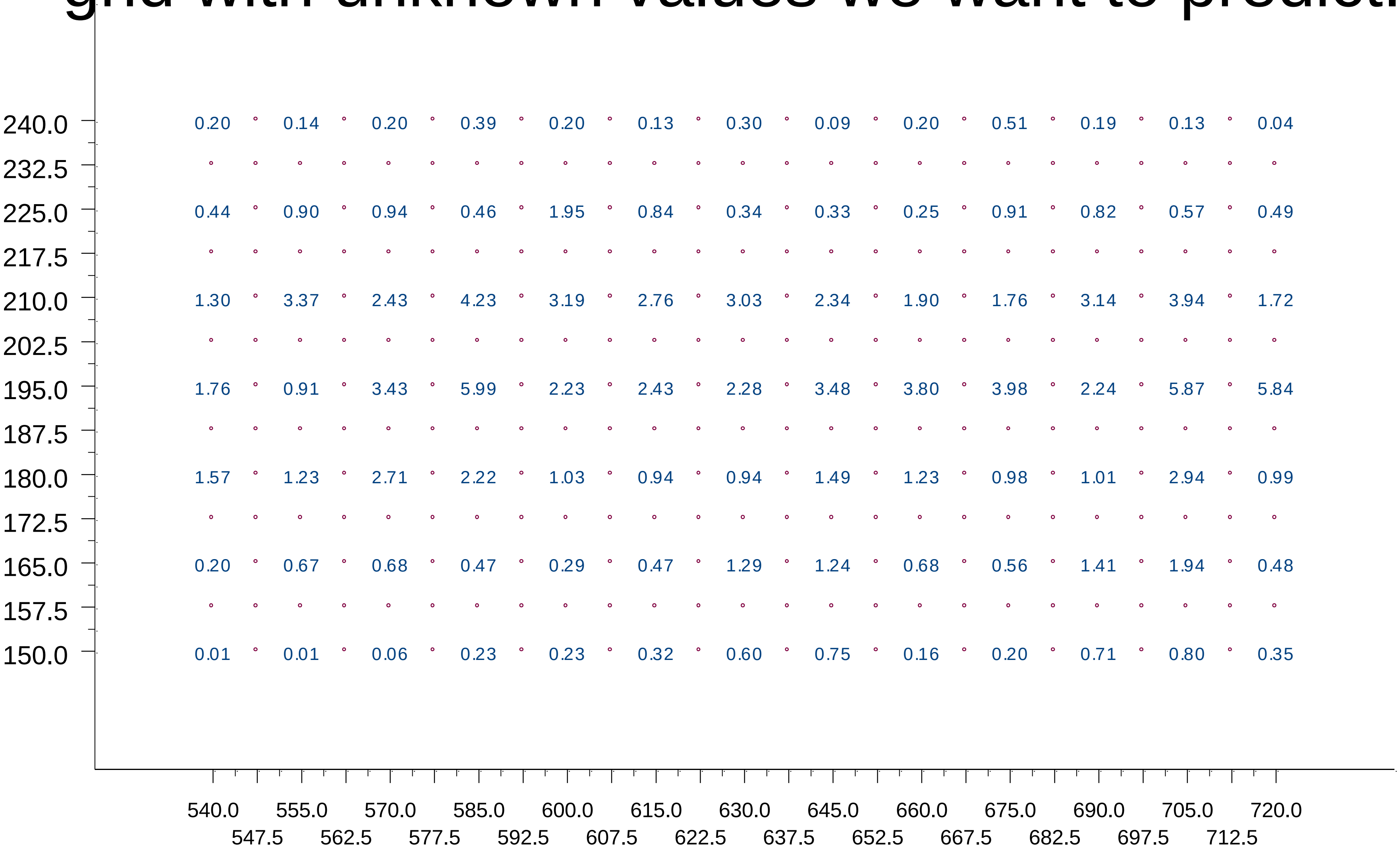
- <u>Sill:</u> The value of the average squared difference at the range.

- <u>Nugget:</u> The vertical jump between zero and the first variogram value.

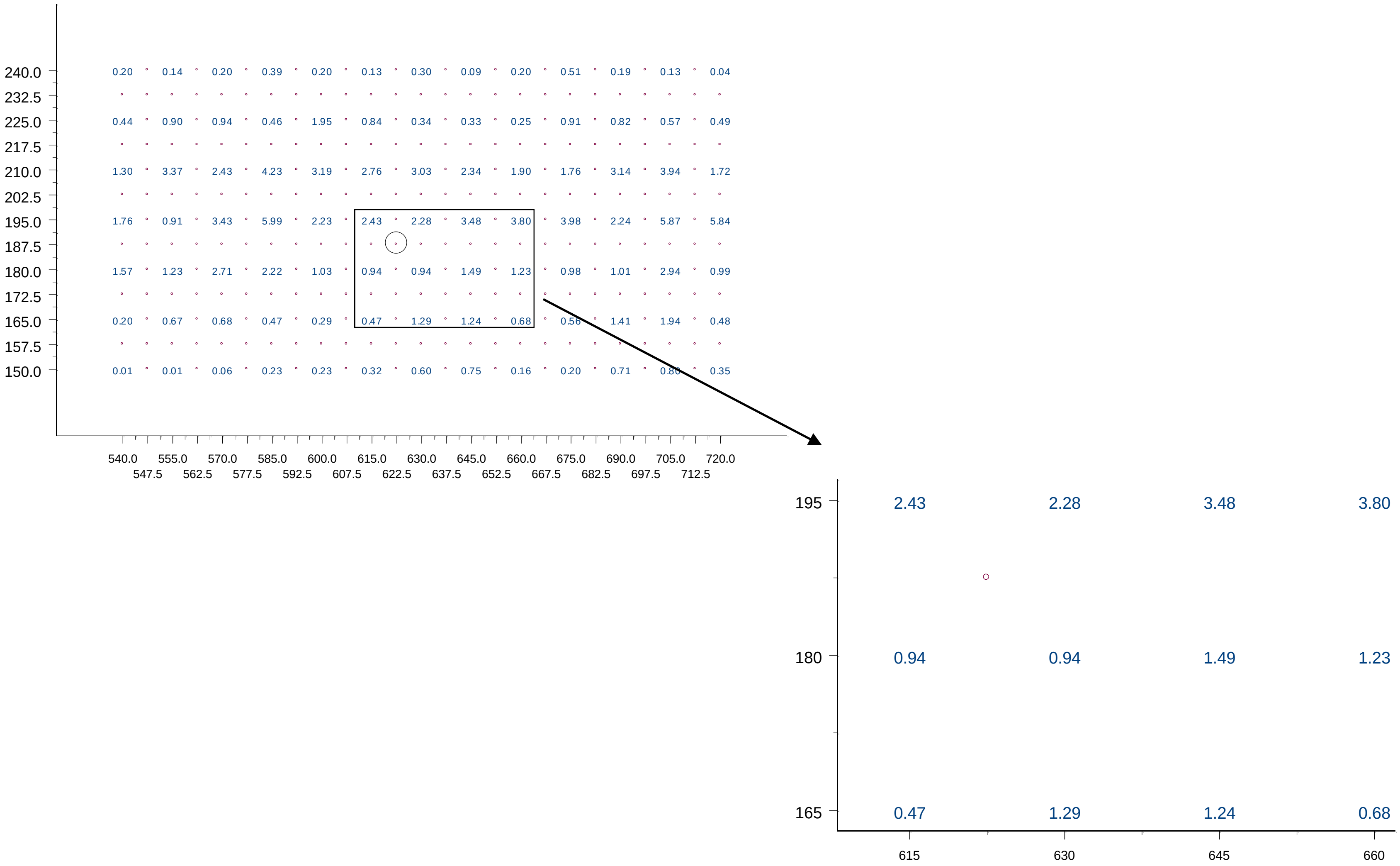


Now that we have our range, sill and nugget we will look at some sample data to calculate the weights.

Example of gpc values in their position on the grid with unknown values we want to predict.



For the purpose of this explanation we will only focus on 12 observed gpc values to predict one unknown value. The picture to the right shows the area we will focus on inside the box.



Step 2: Use the variogram to estimate the covariance at different distances.

Below is a table showing the position of the known values, the corresponding gpc values and the distance to the unknown values.

	X	Y	Z	Distance from unknown value in feet
1	615	165	0.468742	23.72
2	615	180	0.937485	10.61
3	615	195	2.432393	10.61
4	630	165	1.292209	23.72
5	630	180	0.937485	10.61
6	630	195	2.280368	10.61
7	645	165	1.241534	31.82
8	645	180	1.494908	23.72
9	645	195	3.483896	23.72
10	660	165	0.68411	43.73
11	660	180	1.228865	38.24
12	660	195	3.800614	38.24

Next, determine all the distances between known and unknown values, shown in this table.

	0 unknown value	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0.00	23.72	10.61	10.61	23.72	10.61	10.61	31.82	23.72	23.72	43.47	38.24	38.24
1	23.72	0.00	15.00	30.00	15.00	21.21	33.54	3.00	33.54	46.23	45.00	46.23	54.08
2	10.61	15.00	0.00	15.00	21.21	15.00	21.21	33.54	30.00	33.54	47.43	45.00	47.43
3	10.61	30.00	15.00	0.00	33.54	21.21	15.00	42.43	33.54	30.00	54.08	47.43	45.00
4	23.72	15.00	21.21	33.54	0.00	15.00	30.00	15.00	21.21	33.54	30.00	33.54	42.43
5	10.61	21.21	15.00	21.21	15.00	0.00	15.00	21.21	15.00	21.21	33.54	30.00	33.54
6	10.61	33.54	21.21	15.00	30.00	15.00	0.00	33.54	21.21	15.00	42.43	33.54	30.00
7	31.82	30.00	33.54	42.43	15.00	21.21	33.54	0.00	15.00	30.00	15.00	21.21	33.54
8	23.72	33.54	30.00	33.54	21.21	15.00	21.21	15.00	0.00	15.00	33.54	21.21	15.00
9	23.72	42.43	33.54	30.00	33.54	21.21	15.00	30.00	15.00	0.00	33.54	21.21	15.00
10	43.47	45.00	47.43	54.08	30.00	33.54	42.43	15.00	33.54	33.54	0.00	15.00	30.00
11	38.24	47.43	45.00	47.43	33.54	30.00	33.54	21.21	21.21	21.21	15.00	0.00	15.00
12	38.24	54.08	47.43	45.00	42.43	33.54	30.00	33.54	15.00	15.00	30.00	15.00	0.00

$$\mathrm{Cov}(h) = \begin{cases} sill & if \ \ |h| = 0 \\ (sill - nugget)e^{\left(\frac{-3|h|}{range}\right)} & if \ \ |h| > 0 \end{cases}$$

Cov(h) means the covariance at distance h

We will use a range = 430.1663057, nugget = 0.1224344, sill = 0.8386289 from the variogram we created earlier.

We calculate all the covariances at those distances from the table in the previous slide.

Looking at the first column from the previous table we get:

C(00.00) =	sill = 0.8386289
C(23.72) =	$0.7161945e^{\frac{-3}{430.1663057}(23.72)} = 0.606991467$
C(10.61) =	$0.7161945e^{-.006974(10.61)} = 0.665113126$
C(10.61) =	0.665113126
C(23.72) =	0.606991467
C(10.61) =	0.665113126
C(10.61) =	0.665113126
C(31.82) =	0.573660412
C(23.72) =	0.606991467
C(23.72) =	0.606991467
C(43.47) =	0.528895018
C(38.24) =	0.548542205
C(38.24) =	0.548542205

Our new table of covariances…

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.8386	0.6070	0.6651	0.6651	0.6070	0.6651	0.6651	0.5737	0.6070	0.6070	0.5289	0.5485	0.5485
1	0.6070	0.8386	0.6451	0.5810	0.6451	0.6177	0.5668	0.7014	0.5668	0.5188	0.5233	0.5188	0.4912
2	0.6651	0.6451	0.8386	0.6451	0.6177	0.6451	0.6177	0.5668	0.5810	0.5668	0.5145	0.5233	0.5145
3	0.6651	0.5810	0.6451	0.8386	0.5668	0.6177	0.6451	0.5327	0.5668	0.5810	0.4912	0.5145	0.5233
4	0.6070	0.6451	0.6177	0.5668	0.8386	0.6451	0.5810	0.6451	0.6177	0.5668	0.5810	0.5668	0.5327
5	0.6651	0.6177	0.6451	0.6177	0.6451	0.8386	0.6451	0.6177	0.6451	0.6177	0.5668	0.5810	0.5668
6	0.6651	0.5668	0.6177	0.6451	0.5810	0.6451	0.8386	0.5668	0.6177	0.6451	0.5327	0.5668	0.5810
7	0.5737	0.5810	0.5668	0.5327	0.6451	0.6177	0.5668	0.8386	0.6451	0.5810	0.6451	0.6177	0.5668
8	0.6070	0.5668	0.5810	0.5668	0.6177	0.6451	0.6177	0.6451	0.8386	0.6451	0.5668	0.6177	0.6451
9	0.6070	0.5327	0.5668	0.5810	0.5668	0.6177	0.6451	0.5810	0.6451	0.8386	0.5668	0.6177	0.6451
10	0.5289	0.5233	0.5145	0.4912	0.5810	0.5668	0.5327	0.6451	0.5668	0.5668	0.8386	0.6451	0.5810
11	0.5485	0.5145	0.5233	0.5145	0.5668	0.5810	0.5668	0.6177	0.6177	0.6177	0.6451	0.8386	0.6451
12	0.5485	0.4912	0.5145	0.5233	0.5327	0.5668	0.5810	0.5668	0.6451	0.6451	0.5810	0.6451	0.8386

Step 3. Use the covariances to calculate kriging weights.

First well separate the covariance associated with the unknown value. This is in the first column or first row.

We'll call this matrix **D =**

0.6070
0.6651
0.6651
0.6070
0.6651
0.6651
0.5737
0.6070
0.6070
0.5289
0.5485
0.5485
1

The remaining matrix we'll call **C** =

0.6070	0.6651	0.6651	0.6070	0.6651	0.6651	0.5737	0.6070	0.6070	0.5289	0.5485	0.5485	
0.8386	0.6451	0.5810	0.6451	0.6177	0.5668	0.7014	0.5668	0.5188	0.5233	0.5188	0.4912	1
0.6451	0.8386	0.6451	0.6177	0.6451	0.6177	0.5668	0.5810	0.5668	0.5145	0.5233	0.5145	1
0.5810	0.6451	0.8386	0.5668	0.6177	0.6451	0.5327	0.5668	0.5810	0.4912	0.5145	0.5233	1
0.6451	0.6177	0.5668	0.8386	0.6451	0.5810	0.6451	0.6177	0.5668	0.5810	0.5668	0.5327	1
0.6177	0.6451	0.6177	0.6451	0.8386	0.6451	0.6177	0.6451	0.6177	0.5668	0.5810	0.5668	1
0.5668	0.6177	0.6451	0.5810	0.6451	0.8386	0.5668	0.6177	0.6451	0.5327	0.5668	0.5810	1
0.5810	0.5668	0.5327	0.6451	0.6177	0.5668	0.8386	0.6451	0.5810	0.6451	0.6177	0.5668	1
0.5668	0.5810	0.5668	0.6177	0.6451	0.6177	0.6451	0.8386	0.6451	0.5668	0.6177	0.6451	1
0.5327	0.5668	0.5810	0.5668	0.6177	0.6451	0.5810	0.6451	0.8386	0.5668	0.6177	0.6451	1
0.5233	0.5145	0.4912	0.5810	0.5668	0.5327	0.6451	0.5668	0.5668	0.8386	0.6451	0.5810	1
0.5145	0.5233	0.5145	0.5668	0.5810	0.5668	0.6177	0.6177	0.6177	0.6451	0.8386	0.6451	1
0.4912	0.5145	0.5233	0.5327	0.5668	0.5810	0.5668	0.6451	0.6451	0.5810	0.6451	0.8386	1
1	1	1	1	1	1	1	1	1	1	1	1	0

The equation to determine kriging weights is:

C * kriging weights matrix = **D**

where we want the kriging weights matrix.

The easiest way to solve this equation is by using linear algebra.

We can multiply both sides by **C** inverse…

= $\mathbf{C}^{-1} * \mathbf{C} * \text{weights} = \mathbf{D} * \mathbf{C}^{-1}$

= $\mathbf{I} * \text{weights} = \mathbf{D} * \mathbf{C}^{-1}$

= $\text{weights} = \mathbf{D} * \mathbf{C}^{-1}$

$$C^{-1} =$$

4.083	-1.216	-0.461	-0.722	-0.294	-0.137	-2.655	0.458	0.325	0.523	0.161	-0.064	0.139
-1.304	4.506	-1.239	-0.657	-0.842	-0.478	0.660	-0.267	-0.176	-0.232	-0.031	0.061	0.091
-0.441	-1.255	3.914	-0.132	-0.461	-1.228	0.286	-0.075	-0.394	-0.045	-0.015	-0.154	0.165
-1.280	-0.507	-0.066	4.425	-0.806	-0.045	-0.297	-0.695	-0.087	-0.658	-0.160	0.175	0.061
-0.452	-0.802	-0.441	-0.819	4.971	-0.781	-0.066	-0.862	-0.425	-0.258	-0.170	0.105	-0.023
-0.076	-0.500	-1.234	-0.074	-0.790	4.557	0.053	-0.375	-1.097	-0.039	-0.126	-0.299	0.054
-0.359	-0.082	0.039	-1.066	-0.334	0.016	4.924	-1.246	-0.044	-1.465	-0.552	0.170	0.053
-0.099	-0.091	-0.014	-0.534	-0.804	-0.363	-1.142	5.029	-0.748	0.411	-0.366	-1.279	0.010
0.048	-0.087	-0.364	0.006	-0.393	-1.093	-0.047	-0.735	4.548	-0.200	-0.570	-1.115	0.051
-0.186	-0.010	0.032	-0.461	-0.186	-0.023	-1.298	0.402	-0.221	3.875	-1.359	-0.565	0.162
0.007	0.019	0.001	-0.111	-0.153	-0.123	-0.543	-0.361	-0.571	-1.351	4.415	-1.229	0.080
0.058	0.023	-0.167	0.145	0.093	-0.302	0.126	-1.274	-1.110	-0.560	-1.227	4.195	0.158
0.169	0.085	0.160	0.074	-0.020	0.051	-0.045	0.032	0.063	0.190	0.087	0.153	-0.599

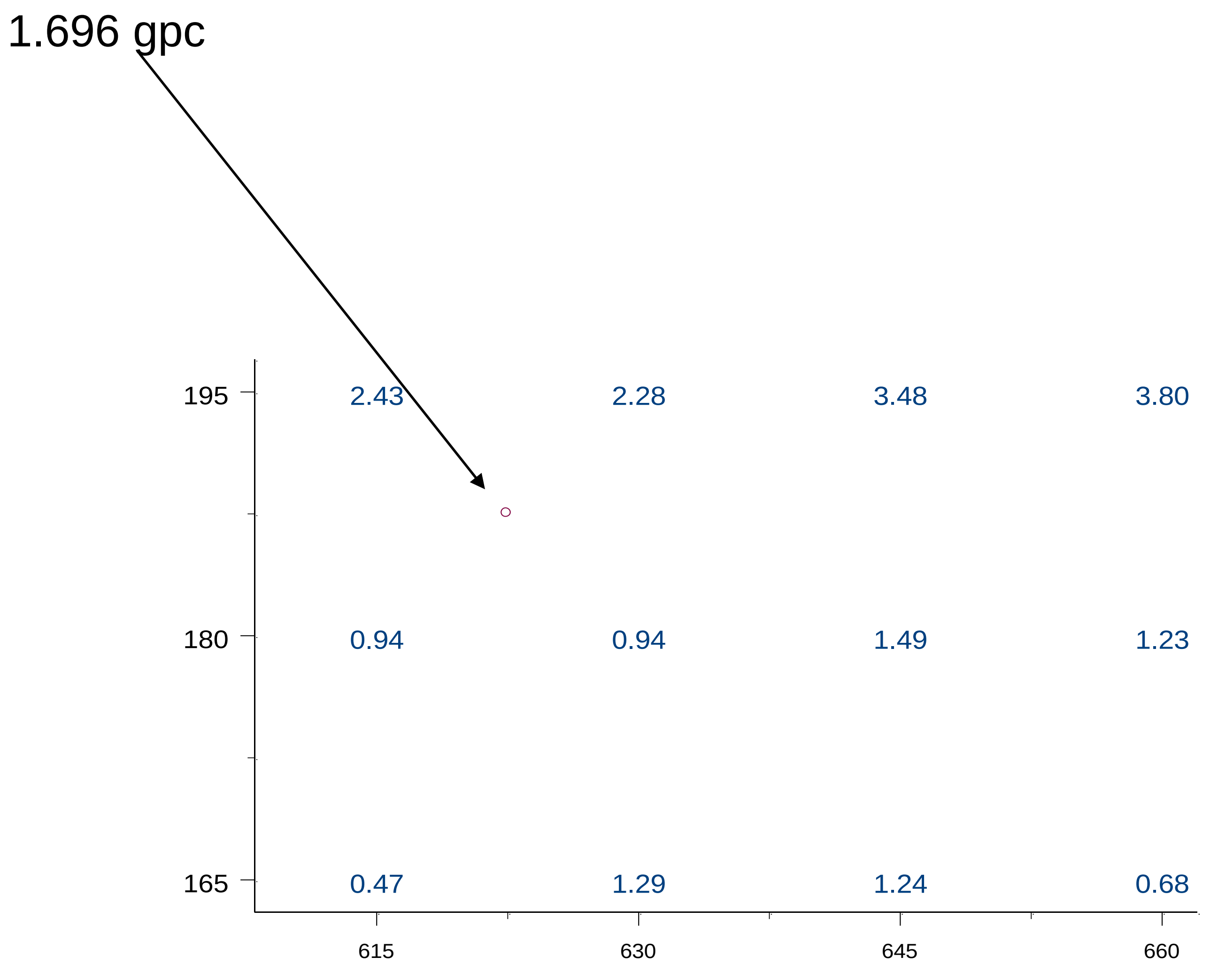
Kriging weights =

0.0580
0.1989
0.2249
0.0384
0.1745
0.1984
0.0043
0.0378
0.0542
0.0045
0.0029
0.0034

Step 4. Calculate the weighted average of the raw data to find the weighted average.

	X	Y	Z	Distance from unknown value (feet)	Weights	Product
1	615	165	0.468742	23.72	0.058	0.027164915
2	615	180	0.937485	10.61	0.199	0.186443159
3	615	195	2.432393	10.61	0.225	0.546960529
4	630	165	1.292209	23.72	0.038	0.049666153
5	630	180	0.937485	10.61	0.175	0.163602356
6	630	195	2.280368	10.61	0.198	0.452333773
7	645	165	1.241534	31.82	0.004	0.005331766
8	645	180	1.494908	23.72	0.038	0.056440308
9	645	195	3.483896	23.72	0.054	0.188875433
10	660	165	0.68411	53.05	0.004	0.003056074
11	660	180	1.228865	38.24	0.003	0.003582303
12	660	195	3.800614	38.24	0.003	0.012744684
						1.696201453

Notice the greater the distance the smaller the weight.


1.696 gpc
195
180
165
2.43
2.28
3.48
3.80
0.94
0.94
1.49
1.23
0.47
1.29
1.24
0.68
615
630
645
660

Re-cap:

- Model the variation across the raw data.
- Use the features of the model to calculate kriging weights.
- Calculate the weighted average of the raw data.